A New Multi-target State Estimation Algorithm for PHD Particle Filter

Lingling Zhao, **Peijun Ma**, **Xiaohong Su**, **Hongtao Zhang**
School of Computer Science and Technology
Harbin Institute of Technology
Harbin, China.
zhaolinglinghit@126.com

Abstract – *Probability hypothesis density (PHD) filter is a new practical method to solve the unknown time-varying multi-target tracking problem. Particle filter implementation of the PHD filter has demonstrated a feasible suboptimal method for tracking multi-target in real-time. To obtain the target states, the peak-extraction from the posterior PHD particles needs to be implemented. A new state estimation method is proposed in this paper, which doesn't need to extract the PHD peaks. The method provides a single-target PHD expression derived from the updated PHD equation. The single-target PHD is approximated by the particles and their weights relevant to the observation. Thus the target states can be directly estimated from the single-target PHD sequentially. Simulation results demonstrate that the new algorithm provides more accurate state estimations and is more efficient than the traditional multi-target state estimation methods such as* k*-means clustering algorithm.*

Keywords: multi-target tracking, PHD particle filter, multi-target state estimation.

1 Introduction

Multi-target tracking is a challenging state estimation problem when the number of targets is time-varying and unknown, observations are corrupted by noise and some of them may be false alarms due to clutter. The positions of the targets can be estimated by assigning a separate filter to each target [1], and a model-data association [2] is needed to assign the measurements to their corresponding targets. Another method is to represent the full joint distribution of targets through Bayesian approaches, which is often computationally expensive, especially when the number of targets is large. Probability hypothesis density (PHD) [3] devised by Mahler is an alternative to solve the problem by propagating the first moment of the joint distribution instead of the joint distribution itself, and meanwhile avoids data association because no target identity information is required to keep.

The interval of the PHD over any region S gives the estimated number of targets that are in so that the PHD can directly estimate the number of targets. However, in general, the PHD recursion equations do not admit a closed-form solution. Two approaches have been provided to the implementation of the PHD filter, i.e. the particle filter and its variations for the implementation of the PHD filter [4-7] and the Gaussian Mixture PHD (GM-PHD) filter [8, 9]. In the particle-filter implementation, the PHD is represented by a set of particles and their corresponding weights propagated forward at each iteration step according to the given state-space model.

Since the PHD filter only produces an intensity function estimates, peaks of the PHD need to be extracted as the target states. The k-means clustering and expectation-maximization (EM) algorithm [10] have been proposed to extract the target locations from the PHD. However, EM algorithm, in which the posterior particle distribution is approximated by a mixture of Gaussians has been found to have poor performance and high computational complexity. Another peak extraction technique has been devised by M Tobias [11], and it exploits the property of the PHD that it integrates to the expected number of targets. This algorithm has a comparable accuracy with the k-means clustering algorithm.

This paper proposes a new multi-target state estimation method for the PHD particle filter. Different from the usual peak-extraction algorithms and clustering analysis technique, the proposed algorithm divides the estimated PHD into several sub PHDs, called single-target PHD, in weight domain. Furthermore, target positions are estimated sequentially from these single-target PHDs instead of PHD. This method provides better performance than the usual multi-target state estimation algorithms for the PHD particle filter, such as the k-means and Tobias' peak extraction algorithm.

The remainder of the paper is organized as follows. In Section 2 we give a general background on the particle PHD filter, and peak extraction algorithm is represented in section 3, simulations and results are provided in section 4. Finally, section 5 concludes the paper.

2 Background

This section provides a formulation of the PHD filter and its implementation based on particle filter for multi-target tracking.

2.1 Multiple target filtering model

In multi-target tracking problem, the number of targets in surveillance region is usually time-varying and unknown; meantime, their states and observations evolve in time, too. Therefore, the states of T_k tracked targets at time k can be naturally represented as a random set $\Gamma_k = \{x_{k,1}, x_{k,2}, ..., x_{k,T_k}\}$, where $x_{k,i}$ is the state of an individual target. Similarly, m_k measurements can be given by a random set $Z_k = \{z_{k,1}, z_{k,2}, ..., z_{k,M_k}\}$, where $z_{k,i}$ is an observation from a target or due to clutter. Then the goal of multi-target filtering is to estimate the target states and the number T_k at time step k based on the observation collection $Z_{1:k} = \{Z_1, Z_2, ..., Z_k\}$.

2.2 PHD filter

The PHD is the first-order moment of multi-target posterior density. Let $D_{k|k}$ denote the PHD. The PHD filter involves a prediction step and an update step, and the prediction operator is defined by

$$D_{k|k-1} = \gamma_k(x) + \int \phi_{k|k-1}(x, x_{k-1}) D_{k-1|k-1} dx_{k-1} \tag{1}$$

where γ_k is the intensity function of a newborn targets and $\phi_{k|k-1}(x, x_{k-1}) = P_S(x_{k-1}) f_{k|k-1}(x, x_{k-1}) + b_{k|k-1}(x, x_{k-1})$, with b_k denoting the PHD of a spawned target, P_S denoting the probability of target survival and $f_{k|k-1}(x|x_{k-1})$ denoting the transition probability density of the single-target motion [4]. $D_{k|k}$is updated by the following equation:

$$D_{k|k} = \Big[\upsilon(x) + \sum_{z \in Z_k} \frac{\psi_{k,z}(x)}{\kappa_k(z) + \langle D_{k|k-1}, \psi_{k,z} \rangle}\Big] D_{k|k-1} \tag{2}$$

In (2),$\upsilon(x) = 1 - P_D(x)$, with $P_D(x)$ denoting the probability of detection. $\psi_{k,z} = P_D(x) g(z|x)$, where $g(z|x)$ is the single-target likelihood function. $\kappa_k(z)$ is the PHD of the clutter RFS and $\kappa_k(z) = \lambda_k c_k(z)$, which means that the clutter points in the surveillance region follow a probability distribution $c_k(z)$and the average number of them per scan is λ_k.Finally the notation $\langle D(k|k-1), \psi_{k,z} \rangle = \int D_{k|k}(x_t|Z_{1:t}) \varphi(x_k) d(x_k)$.

With the new measurements received, the PHD is propagated forwards recursively. But in the prediction step and update step there are still multiple integrals without closed-form expressions in common [5], so particle filter is exploited for the implementation of the PHD.

2.3 PHD particle filter

A particle filter implementation of the PHD filter was devised in [5]. The algorithm can be briefly described by the following stage. Here the PHD is represented by a collection of particles and their associate weights $\{x_k^i, w_k^i\}_{i=1}^{L_k}$, where k is the time step and L_k denotes the number of particles at time k. Suppose that there are L_{k-1} particles and weights $\{x_{k-1}^i, w_{k-1}^i\}_{i=1}^{L_{k-1}}$ at time $k-1$, which are predicted forward to time k, and J_k new particles are added to describe the PHD of the newborn targets. When the new measurements are received, the particle weights are updated and the PHD at time k is given by $\{\tilde{x}_k^i, \tilde{w}_k^i\}_{i=1}^{L_{k-1}+J_k}$. The number of targets can be estimated by $\hat{T}_k = round(\sum_{i=1}^{L_{k-1}+J_k} \tilde{w}_k^i)$, where $round(\sum_{i=1}^{L_{k-1}+J_k} \tilde{w}_k^i)$ is the nearest integer to $\sum_{i=1}^{L_{k-1}+J_k} \tilde{w}_k^i$. When implementing the resampling step, note that the new weights $\{w_k^i\}_{i=1}^{L_k}$ sum to $\hat{N}_{k|k}$ instead of 1 and the number of new particles $L_k = \hat{T}_k \cdot \rho$, where ρ is the number of particles assigning for a target. L_k is not always equal to $L_{k-1} + J_k$. The k-means or EM algorithms are used for the state estimation based on the particles after the resampling step in common.

3 Multi-target state estimation

This section provides a novel multi-target state estimation based on the particle representation of the PHD. The idea is to decompose the particle weights according to observations at each iteration and estimate each target state independently. In section 3.1, a brief review is given on PHD state estimation methods.

3.1 Multi-target state estimation based on cluster analysis

In resampling step of the particle PHD filter, the particles with low weights are eliminated and particles with high weights are multiplied, thus, the new particles focus on the important zones of the space. Cluster analysis techniques, such as the k-means algorithm, are implemented to separate these particles into homogeneous clusters. In this method, a cluster denotes a subset of particles associated with a same target, and the centers of the clusters denote the estimated target states. The k-means algorithm can provide reliable estimates only when the particles associated with different targets are dispersive to each other and the initial cluster centers are reasonable.

EM algorithm is another cluster analysis method used to extract the target locations from the PHD in the literature [10]. It fits the PHD by a mixture of Gaussians and considers the mean and the covariance of a Gaussian as an estimate of target state. It is found to have the following problems: high computational cost, singular covariance matrix output sometimes and low tracking accuracy when the PHD does not accord to a Gaussian mixture distribution.

Different from clustering analysis techniques, Tobias' peak extraction algorithm makes use of the integral properties of the PHD. In Tobias' peak extraction algorithm, peaks are extracted directly from the approximated PHD, i.e. samples and their associate weights $\{\tilde{x}_k^i, \tilde{w}_k^i\}_{i=1}^{L_{k-1}+J_k}$.

3.2 Multi-target state estimation based on single-target PHD

PHD filter estimates multi-target state by extracting multiple peaks from the PHD. It is substantially to decompose the estimated PHD into several parts, each of which is regarded as a posterior density of a single target. EM algorithm and the k-means algorithm decompose PHD only depending on the distribution of particles and the property of the PHD itself is not considered at all. EM, the k-means and Tobias' peak extraction algorithm estimate targets according to the same principle: partition the PHD that is approximated by $\{\tilde{x}_k^i, \tilde{w}_k^i\}_{i=1}^{L_{k-1}+J_k}$ into sub-groups, and then extract target states from these sub-groups.

3.2.1 single-target PHD

We propose a novel partition method which is different from the above mentioned algorithms. Before discuss this method, we give the description of the single-target PHD.

Suppose T_{k-1} targets exist in the surveillance region at time step $k-1$, and the PHD at this scan is $D_{k-1|k-1}(x|Z_{1:k-1})$, which is approximated by particles and their corresponding weights $\{\tilde{x}_{k-1}^i, \tilde{w}_{k-1}^i\}_{i=1}^{L_{k-1}}$. Assume that M_k observations are obtained and T_k targets exist in the surveillance region at time k. Via the PHD particle filter, the approximated PHD at time k can be represented by $\{\tilde{x}_k^i, \tilde{w}_k^i\}_{i=1}^{L_{k-1}+J_k}$, where

$$\tilde{w}_k^i = \Big[\upsilon(\tilde{x}_k^i) + \sum_{z_{k,j}\in Z_k} \frac{\psi_{k,z}(\tilde{x}_k^i)}{\kappa_k(z_{k,j}) + C_k(z_{k,j})}\Big]\tilde{w}_{k|k-1}^i \quad (3)$$

Now consider the following scenario. Besides the T_k targets, an additional target exists in the region at this time step k, then, some changes will happen to the PHD at time k.

Firstly, particles $\tilde{x}_k^i$ and their predicted weights $\tilde{w}_{k|k-1}^i$ remain unchanged since no variation appears in the prediction step. In the update step, another observation z_{k,M_k+1} due to the additional target is collected with the detection probability $P_D(x)$. Denote $D_{k|k}^*(x|Z_{1:k})$ as the new PHD, $\tilde{w}_k^{i*}$ as the new posterior weights. If z_{k,M_k+1} is not received by the sensor, the PHD filtering will beunchanged, so $\tilde{w}_{k,1}^{i*} = \tilde{w}_k^i$, and this happens with the probability $\upsilon = 1 - P_D(x)$; otherwise, z_{k,M_k+1} is received by the sensor with the probability $P_D(x)$, in this case,

$$\tilde{w}_{k,2}^{i*} = \Big[\upsilon(\tilde{x}_k^i) + \sum_{z_{k,j}\in Z_k} \frac{\psi_{k,z}(\tilde{x}_k^i)}{\kappa_k(z_{k,j}) + C_k(z_{k,j})}\Big]\tilde{w}_{k|k-1}^i + \frac{g(z_{k,M_k+1}|\tilde{x}_k^i)}{\kappa_k(z_{k,M_k+1}) + C_k(z_{k,M_k+1})}\tilde{w}_{k|k-1}^i \quad (4)$$

and

$$\tilde{w}_k^{i*} = (1 - P_D(\tilde{x}_k^i))\tilde{w}_{k,1}^{i*} + P_D(\tilde{x}_k^i)\tilde{w}_{k,2}^{i*} \quad (5)$$

Thus,

$$\begin{aligned}\tilde{w}_k^{i*} =& \Big\{\Big[\upsilon(\tilde{x}_k^i) + \sum_{z_{k,j}\in Z_k} \frac{\psi_{k,z}(\tilde{x}_k^i)}{\kappa_k(z_{k,j}) + C_k(z_{k,j})}\Big]\tilde{w}_{k|k-1}^i \\ &+ \frac{g(z_{k,M_k+1}|\tilde{x}_k^i)\tilde{w}_{k|k-1}^i}{\kappa_k(z_{k,M_k+1}) + C_k(z_{k,M_k+1})}\Big\}P_D(\tilde{x}_k^i) \\ &+ (1 - P_D(\tilde{x}_k^i))\tilde{w}_k^i \\ =& \Big\{\tilde{w}_k^i + \frac{g(z_{k,M_k+1}|\tilde{x}_k^i)\tilde{w}_{k|k-1}^i}{\kappa_k(z_{k,M_k+1}) + C_k(z_{k,M_k+1})}\Big\}P_D(\tilde{x}_k^i) \\ &+ (1 - P_D(\tilde{x}_k^i))\tilde{w}_k^i \\ =& \tilde{w}_k^i + \frac{g(z_{M_k+1}|\tilde{x}_k^i)\tilde{w}_{k|k-1}^i}{\kappa_k(z_{k,M_k+1}) + C_k(z_{k,M_k+1})}P_D(\tilde{x}_k^i) \\ =& \tilde{w}_k^i + \frac{\psi_{k,z_{k,M_k+1}}(\tilde{x}_k^i)}{\kappa_k(z_{k,M_k+1}) + C_k(z_{k,M_k+1})}\tilde{w}_{k|k-1}^i\end{aligned} \quad (6)$$

Let $\Delta w_k^i = \tilde{w}_k^{i*} - \tilde{w}_k^{i*}$, then

$$\Delta w_k^i = \frac{\psi_{k,z_{k,M_k+1}}(\tilde{x}_k^i)}{\kappa_k(z_{k,M_k+1}) + C_k(z_{k,M_k+1})}\tilde{w}_{k|k-1}^i \quad (7)$$

Therefore, $D_{k|k}^*(x|Z_{1:k})$ has the same supporting sample set$\{\tilde{x}_k^i\}_{i=1}^{L_{k-1}+J_k}$ as $D_{k|k}(x|Z_{1:k})$, but the weight $\tilde{w}_k^{i*}$ of each particle is heavier than $\tilde{w}_k^i$ to a different extent. Let $\Delta D_{k|k}(x|z_{k,M_k+1})$ denote the difference between $D_{k|k}^*(x|Z_{1:k})$ and $D_{k|k}(x|Z_{1:k})$, then

$$\Delta D_{k|k}(x|z_{k,M_k+1}) = \frac{\psi_{k,z_{k,M_k+1}}(\tilde{x})D_{k|k-1}}{\kappa_k(z_{k,M_k+1}) + \langle D_{k|k-1}, \psi_{k,z_{k,M_k+1}}\rangle} \quad (8)$$

Thus, the PHD $D_{k|k}^*(x|Z_{1:k})$ representing T_k targets is equivalent to the sum of PHD $D_{k|k}(x|Z_{1:k})$ representing $T_k - 1$ targets and a posterior density $\Delta D_{k|k}(x|z_{k,M_k+1})$ relevant to a single target, where $\Delta D_{k|k}(x|z_{k,M_k+1})$ is defined as the single-target PHD for the measurement z_{k,M_k+1}.

From (2) and (8), we can derive that

$$D_{k|k}(x|Z_{1:k}) = \sum_{j=1}^{M_k} \Delta D_{k|k}(x|z_{k,j}) + \Delta D_{k|k}(x|\phi) \quad (9)$$

where $\Delta D_{k|k}(x|\phi) = \upsilon(x)D_{k|k-1}$, which denotes the intensity from the targets with no measurements received. If detection probability $P_D(x) = 1$, $\Delta D_{k|k}(x|\phi)$

will be zero. Formula (9) means that the PHD can be regarded as the sum of M_k single-target PHDs and a PHD relevant to the targets without measurements, and each of single-target PHD is relevant to a measurement received at time k. Note that since the measurement $z_{k,j}$ may be due to clutter, the single-target PHD $\Delta D_{k|k}(x|z_{k,j})$ does not always represent a target posterior density, sometimes $\Delta D_{k|k}(x|z_{k,j})$ is from a clutter point. Moreover, since a single-target PHD is relevant to only one observation, if the assume that one observation is from a target or a clutter point is satisfied, it will be impossible that one target is divided into more than one single-target PHDs.

Figure 1 shows the relationship between PHD and its single-target PHDs with a unity probability of detection. At this scenario two targets exist in the surveillance region and three observations are received, one of which are from clutter. Subgraph (a) shows the PHD approximated by 500 particles (represented by the dots), and there are one peak with a nearly plat hat. Three single-target PHDs are shown in subgraph (b), from which we can find that two single-target PHDs are sharp and overlap each other, while the other one is flat and all sub-weights of them are nearly zero. In fact, the sharp two represent the targets present, which are in close proximity that the peaks of them merge as shown in (a). The flat one are from clutter. It is indicated that the target densities are divided naturally by the single-target PHDs.

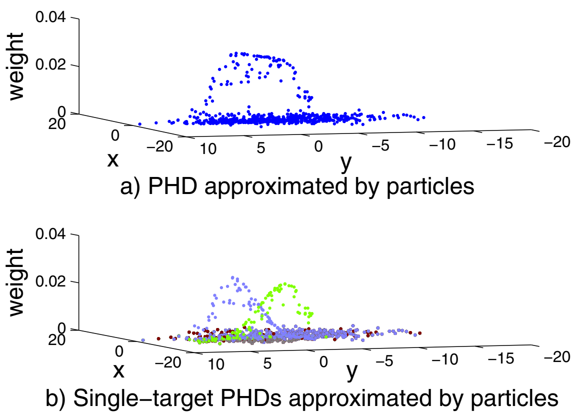

Figure 1: An approximated PHD and its single-target PHDs

3.2.2 Multi-target state estimation algorithm based on single-target PHD

Now we can estimate the target states based on the single-target PHDs. The expected number of targets present $\hat{T}_k$ can be obtained by summing the weights of all posterior PHD particles and rounding to the nearest integer as indicated in PHD particle filter. If the observations received at time step k are corrupted by clutter, the number of single-target PHDs M_k will be larger than $\hat{T}_k$. Therefore, we need to find those single-target PHDs representing the target densities from all current single-target PHDs. First, we sum of the $\tilde{w}_k^{i,j}$ for each observation $z_{k,j}$, where $\tilde{w}_k^{i,j}$ is the sub-weight of the ith particle for observation $z_{k,j}$. Denote the sub-weight sum associated with observation $z_{k,j}$ as W_k^j. Then, we pick up the $\hat{T}_k$ single-target PHDs with the largest W_k^j. These selected single-target PHDs are considered as the individual target densities. Finally the target states can be separately estimated from these $\hat{T}_k$ single-target PHDs represented by all the particles and their corresponding sub-weights. The PHD particle filter with multi-target state estimation based on single-target PHDs is described as follows:

At time $k \geq 1$,

- **Step 1 Prediction**

  For $i = 1, ..., L_{k-1}$, sample $\tilde{x}_k^i$ from an important density $q_k(\cdot|x_{k-1}^i, Z_k)$ and compute the predicted weights

$$\tilde{w}_{k|k-1}^i = \frac{\phi_{k|k-1}(\tilde{x}_k^i, x_{k-1}^i)}{q_k(\tilde{x}_k^i|x_{k-1}^i, Z_k)} w_{k-1}^i \qquad (10)$$

  For $i = L_{k-1} + 1, ..., L_{k-1} + J_k$, sample $\tilde{x}_k^i \sim p_k(\cdot|Z_k)$ and the weights of newborn particles

$$\tilde{w}_{k|k-1}^i = \frac{1}{J_k} \frac{\gamma_k(\tilde{x}_k^i)}{p_k(\tilde{x}_k^i|Z_k)} \qquad (11)$$

- **Step 2 Update**

  For each observation $z_j \in Z_k$,

$$C_k(z_{k,j}) = \sum_{i=1}^{L_{k-1}+J_k} \psi_{k,z_{k,j}}(\tilde{x}_k^i)\tilde{w}_{k|k-1}^i \qquad (12)$$

$$G_k^{i,j} = \frac{\psi_{k,z}(\tilde{x}_k^i)}{\kappa_k(z_{k,j}) + C_k(z_{k,j})} \qquad (13)$$

  For $i = 1, ..., L_{k-1} + J_k$,update weights

$$\tilde{w}_k^i = \Big[v(\tilde{x}_k^i) + \sum_{z_{k,j} \in Z_k} G_k^{i,j}\Big]\tilde{w}_{k|k-1}^i \qquad (14)$$

  Meanwhile, compute the sub-weight of each particle for all observations $z_{k,j}$

$$\tilde{w}_k^{i,j} = G_k^{i,j}\tilde{w}_{k|k-1}^i \qquad (15)$$

  And the particle sub-weight for the targets without measurements obtained is

$$\tilde{w}_k^{i,0} = v(\tilde{x}_k^i)\tilde{w}_{k|k-1}^i \qquad (16)$$

- **Step 3 Resampling.**

  Compute the sum of particle weights $\hat{N}_{k|k} = \sum_{i=1}^{L_{k-1}+J_k} \tilde{w}_k^i$, which is the approximated interval of the PHD over the surveillance region, then the expected number of targets $T_k = round(\hat{N}_{k|k})$. Let the new number of particles L_k after resampling be $\hat{T}_k \cdot \rho$.

  Resample $\{\tilde{x}_k^i, \frac{\tilde{w}_k^i}{\hat{N}_{k|k}}\}_{i=1}^{L_{k-1}+J_k}$ to get $\{x_k^i, w_k^i\}_{i=1}^{L_k}$, and the new weight of each particle is $\frac{\hat{N}_{k|k}}{L_k}$.

- **Step 4 Multi-target state estimation based on single-target PHDs.**

  For each observation $z_{k,j}, j = 1, ..., M_k$, compute the sum of sub-weights W_k^j relevant to $z_{k,j}$,

$$W_k^j = \sum_{i=1}^{L_{k-1}+J_k} \tilde{w}_k^{i,j} \quad (17)$$

  Compute the sum of sub-weight W_k^0 relevant to targets without measurements:

$$W_k^0 = \sum_{i=1}^{L_{k-1}+J_k} \tilde{w}_k^{i,0} \quad (18)$$

  For p = 1 to $\hat{T}_k$,

  find the largest W_k^q, where $q = arg_j max(W_k^j)_{j=0}^{M_k}$. Return $\zeta_{p,k} = w_k^{i,q} \cdot \tilde{x}_k^i$ as the pth estimated target state, where

$$w_k^{i,q} = \frac{\tilde{w}_k^{i,q}}{\sum_{i=1}^{L_{k-1}+J_k} \tilde{w}_k^{i,q}} \quad (19)$$

  and then let $W_k^q = 0$.

Remark: The computational complexity of the proposed algorithm is $O(TN)$, where T is the target number and N is the number of particles, while the k-means algorithm has a time complexity of $O(\tau TN)$, where τ is the number of iterations in the clustering procedure, and EM algorithm is $O(\tau T^2 N)$.

4 Simulation results

For verification purposes, this section demonstrates results on estimating target locations from the estimated PHD. The test scenarios consist of varying unknown number of targets moving in a 2D region $[-100, 100] \times [-100, 100]$, and the sensor location is $(0 - 100)^T$. The trajectories have been simulated according to the following Gaussian model:

$$X_k = \begin{pmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{pmatrix} X_{k-1} + \begin{pmatrix} T^2/2 & 0 \\ T & 0 \\ 0 & T^2/2 \\ 0 & T \end{pmatrix} v_{k-1}$$

where T is the sampling period and $T = 1$. Measurements are generated according to the bearing and range tracking model:

$$r_k = \left\| \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} - \begin{pmatrix} 0 \\ -100 \end{pmatrix} \right\| + n_{1,k}$$

$$\theta_k = \arctan(\frac{y_k}{x_k + 100}) + n_{2,k}$$

where the state vector $X_k = \begin{pmatrix} x_k & \dot{x}_k & y_k & \dot{y}_k \end{pmatrix}$, v_k, $n_{1,k}$, and $n_{1,k}$ are independent zero-mean Gaussian white noise. Suppose that four targets appear at the random time steps over time and their positions follow the intensity function $p_k = N(\cdot, \bar{x}, Q)$, where

$$\bar{x} = \begin{pmatrix} 0 \\ 3 \\ 0 \\ -3 \end{pmatrix}, Q = \begin{pmatrix} 10 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Clutter follows a uniform distribution with average r clutters per scan in the surveillance region. Let the important sampling density $q_k = f_{k|k-1}$, where $f_{k|k-1}$ is the state transfer function. Assign 200 particles to an exist or newborn target. 400 particles are still maintained when the expected number of targets is zero. For simplicity, let detection probability $P_D(x) = 1$.

In order to assess the performance of the proposed single-target PHD (STPHD) algorithm, we implement the PHD particle filters with different average clutter rates, respectively. Within the iteration of the PHD particle filter, the standard k-means peak extraction (KPE), Tobias' peak extraction (TPE) algorithm and the proposed algorithm is run on the same estimated PHD to obtain the target states.

The simulated trajectories of four targets are shown in Fig. 2. and 3. To assess the performance of these

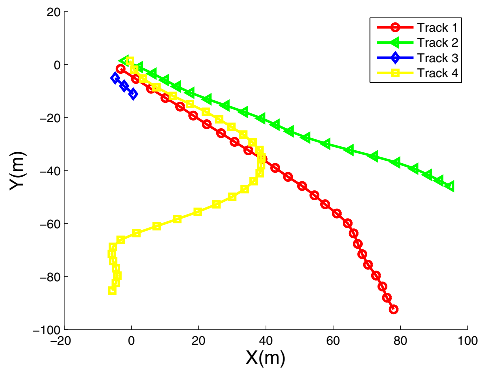

Figure 2: Actual trajectories of four targets

algorithms, the Wasserstein distance [12] between the positions estimates of the multi-target state and ground truth at each time step is used as a multi-target miss-distance. Munkres algorithm [13] is exploited to compute the Wasserstein distance. Moreover, the run time

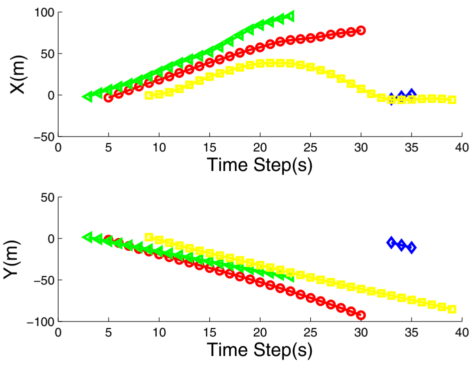

Figure 3: X and Y components of four true tracks

has been measured for the three estimation algorithms. The program was run on a PC having 2.28GHz Intel Core 2 Duo processor and 3G of main memory.

4.1 Example 1

To compare the performance of the proposed algorithm with the other two peak extraction algorithms when the approximated PHD provides the correct estimated number of targets, we suppose there is no clutter (clutter rate $r = 0$) in this test scenario and implement the PHD particle filter with these state estimation algorithms for 50 independent Monte Carlo runs. Note that all these 50 runs have the correct estimated number of targets in 40 iterations. Figure 4 shows the average Wasserstein multi-target miss-distance of the three state estimation algorithms. It is clear that the proposed algorithm has the significant lower Wasserstein miss-distance and provides fewer spurious estimates than the other two algorithms. Fig. 5 shows

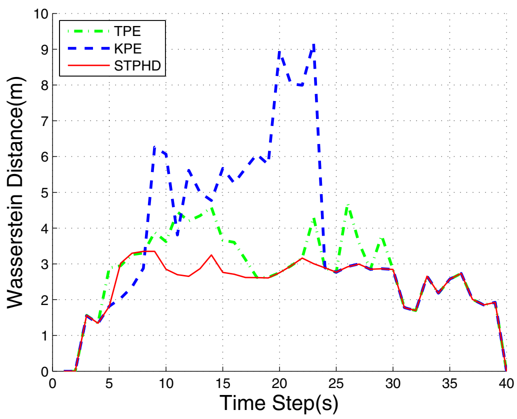

Figure 4: Wasserstein miss-distance over 50 Monte Carlo runs at $r = 0$

the true number is just the same as the estimated target numbers. Fig. 2, 3 and 5 display that three targets

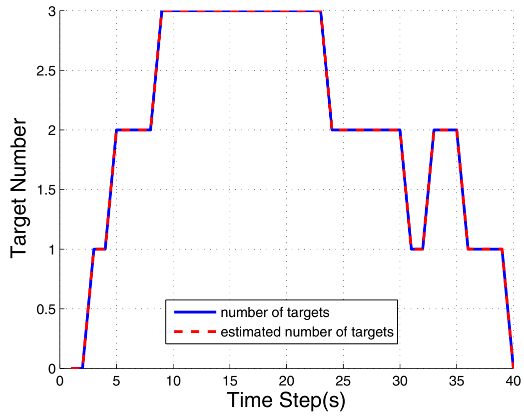

Figure 5: Actual against estimated number of targets

exist in the surveillance region and are in close proximity from time 9 to 23, when the k-means algorithm has a significant larger Wassertein miss-distance than the other two algorithms.

The average results in terms of Wasserstein miss-distance and run time of each estimation algorithm are listed in table 1.

Table 1: Average results over 50 Monte Carlo runs with $r = 0$

Algorithm	Miss distance error		Run time(second)
	Mean	Variance	
TPE	3.0269	0.2274	0.1900
KPE	3.9360	0.6159	0.2192
STPHD	2.6152	0.1079	0.0576

4.2 Example 2

In this example, we consider the scenario when observations are corrupted by clutters. Let the average clutter rate r=10 points per scan, so some observations may be due to clutter and the expected number of targets is incorrect at some observing time. Fig. 6 displays the error of estimated target number E_t, which is calculated by:

$$E_k = \frac{1}{R}\sum_{j=1}^{R} \|T_{k,j} - N_{k,j}\| \tag{20}$$

where $T_{k,j}$ is the true number of targets at time step k of the jth Monte Carlo run, while $N_{k,j}$ is the corresponding estimated target number. And R is the number of Monte Carlo runs and $R = 50$. Fig. 7 shows the Wasserstein miss-distance averaging by 50 independent Monte Carlo runs.

Similar to example 1, the proposed STPHD algorithm has the lowest miss-distance. Figs. 6-7 show that both the Wasserstein miss-distance and the error of estimated target number increase with the clutter

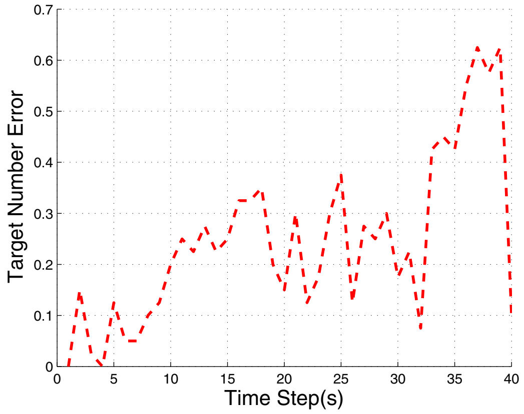

Figure 6: Error of estimated target number over 50 Monte Carlo runs at $r = 10$

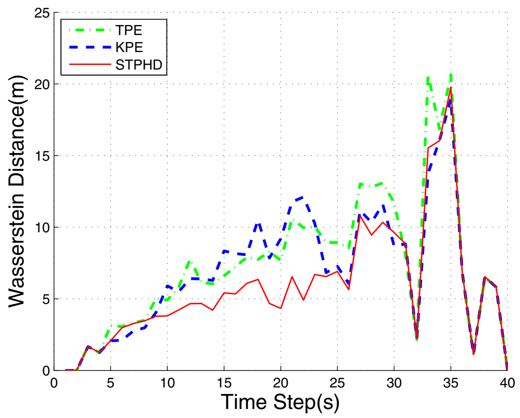

Figure 7: Wasserstein miss-distance over 50 Monte Carlo runs at $r = 10$

rate. To assess the performance of the proposed algorithm in denser clutter, let the average clutter rate $r = 50$. In this case, Figs. 8-9 show an increased error in the estimated target number and the Wasserstein miss-distance. Again STPHD provides better target positions estimates from table 2 and fig. 9, however, KPE, TPE, and the proposed STPHD all become unreliable because of the increased average number of clutter points.

The average time taken on state estimation using these three algorithms is listed in tables 1-3. When the estimated target number is zero, the run time is zero too, since no estimation algorithms are called. Whatever the clutter rate r is, the computation time of the proposed algorithm is much less than the other two algorithms. Overall, the proposed STPHD outperforms the k-means clustering analysis technique when used to obtain the multi-target states from the estimated PHD.

Table 2: Average results over 50 Monte Carlo runs with $r = 10$

Algorithm	Miss distance error		Run time(second)
	Mean	Variance	
TPE	8.4618	5.5992	1.1604
KPE	7.9756	4.6468	0.1269
STPHD	6.7431	5.5667	0.0210

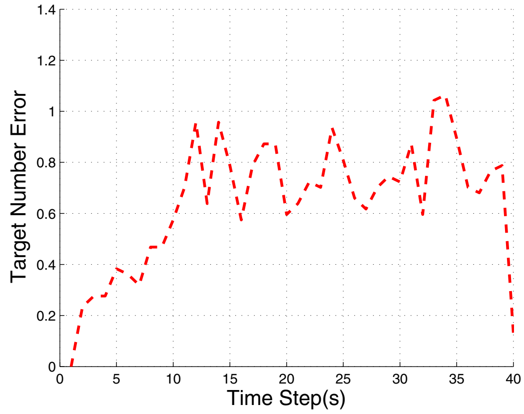

Figure 8: Error of estimated target number over 50 Monte Carlo runs at $r = 50$

5 Conclusions

This paper proposes a new multi-target state estimation algorithm for PHD particle filter. This method decomposes the estimated PHD into several single-target PHDs, each of which is relevant to an observation obtained. All the single-target PHDs have the same sample set as the estimated PHD, and the sum of their sample weights is equal to the estimated PHD, too. In this method, target positions are obtained from these single-target PHDs in weight domain instead of PHD in sample domain. Simulation results indicate that the proposed algorithm works more accurately and efficiently than clustering technique.

Acknowledgment

This paper was supported by the National Natural Science Foundation of China (NSFC) under Grant 60773067.

Table 3: Average results over 50 Monte Carlo runs with $r = 50$

Algorithm	Miss distance error		Run time(second)
	Mean	Variance	
TPE	16.2916	13.1440	1.5373
KPE	14.6893	11.6826	0.3051
STPHD	13.8128	14.3689	0.0574

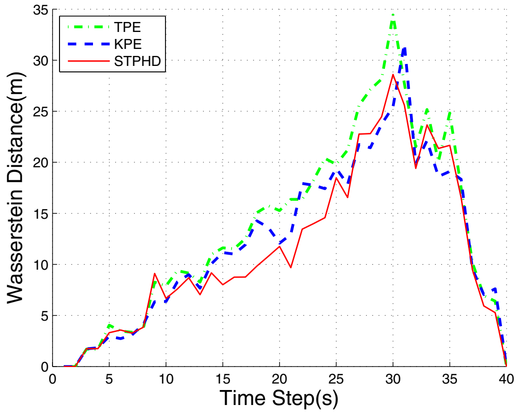

Figure 9: Wasserstein miss-distance over 50 Monte Carlo runs at $r = 50$

References

[1] H. Sidenbladh. "Multi-target particle filtering for the probability hypothesis density," *Proc. 6th International Conference on Information Fusion, FUSION 2003*, Cairns, Australia, 2003, pp. 800–806.

[2] Y. Bar-Shalom, and E. Tse, "Tracking in a cluttered environment with probabilistic data association," *Automatica*, Vol. 11, No. 5, pp. 451–460, 1975.

[3] R. Mahler, "Multitarget Bayes filtering via first-order multitarget moments," *IEEE Trans. on Aerospace and Electronic Systems*, Vol. 39, No. 4, pp. 1152–1178, 2003.

[4] T. Zajic, and R. Mahler, "A particle-systems implementation of the PHD multitarget tracking filter," *Proc. SPIE Signal Processing, Sensor Fusion, and Target Recognition XII*, pp. 291–299, 2003.

[5] B.Vo, S. Singh, and A. Doucet, "Sequential Monte Carlo implementation of the PHD filter for multi-target tracking," *Proc. 6th International Conference on Information Fusion, FUSION 2003*, Cairns, Australia, 2003, pp. 792–799.

[6] B, Vo, S. Singh, and A. Doucet. "Sequential Monte Carlo Methods for Multi-Target Filtering with Random Finite Sets," *IEEE Trans. on aerospace and electronic systems*, vol. 41, No. 4, pp. 1224–1245, 2005.

[7] M. Vihola, "Rao-Blackwellised particle filter in random set multitarget tracking," *IEEE Trans. on Aerospace and Electronic Systems*, Vol. 43, No. 2, pp. 689–705, 2007.

[8] B. Vo, and W. Ma, "A closed form solution for the probability hypothesis density filter," *Proc. 8th International Conference on Information Fusion, FUSION 2005*, Philadephia, USA, 2005, Vol.2, pp. 856–863.

[9] B. Vo, and W. Ma, "The Gaussian Mixture probability hypothesis density filter," *IEEE Trans. on Signal Processing*, Vol. 54, No. 11, pp. 4091–4104, 2006.

[10] D. E. Clark, J. Bell, and H. Watt. "Multi-target state estimation and track continuity for the particle PHD filter," *IEEE Trans. on Aerospace and Electronic Systems*, Vol. 43, No. 4, pp. 1441–1453, 2007.

[11] M. Tobias*, and A.D. Lanterman. "Techniques for birth-particle placement in the probability hypothesis density particle filter applied to passive radar," *IET Radar Sonar Navigation*, Vol. 2, No. 5, pp. 351–365, 2008.

[12] J. R. Hoffman, and R. P. S. Mahler, "Multitarget miss distance via optimal assignment," *IEEE Trans. on Systems, Man and Cybernetics - Part A: Systems and Humans*, Vol. 34, No.3, pp. 327–336, 2004.

[13] F. Bourgeois, and J. C. Lassalle, "An extension of the Munkres algorithm for the assignment problem to rectangular matrices," *Commun. ACM*, Vol. 14, No. 12, pp. 802–804, 1971.